An Efficient Wavelet-VQ Coder for ECG

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ABSTRACT

A novel Vector Quantisation (VQ) scheme for compression of electrocardiograms (ECG) is proposed in this paper. The ECG stream is segmented into beats, which are then period and amplitude normalised (PAN). The PAN vectors are input to the VQ coder. To reduce the computation complexity, the coder uses a product structured codebook. Wavelet transform is used to decompose the PAN vector into sub-vectors. The wavelet coefficients at each scale are treated as separate entities and coded using independent codebooks. This coder provides a favourable trade-off between the computational complexity and the performance of the VQ.

Keywords: ECG compression, wavelet transform, product code vector quantisation, period and amplitude normalisation.

I. Introduction

The need for compression of electrocardiogram (ECG) data arises due to fact that a large amount of ECG data is generated in some hospitals which needs to be stored for the purposes of creating a database and follow-up of patients. An overview of ECG compression research before 1990 is presented in [1]. More recent work on ECG compression is found in [2]-[5].

A simple consequence of Shannon's rate distortion theory is that better performance can always be achieved by coding vectors instead of scalars, even if the data source is memoryless. This fact leads to designing a coder where vectors instead of scalars are quantised to achieve better compression. This scheme, known as vector quantization (VQ) leads to a low bit rate. These characteristics have led researchers to apply VQ on ECG. VQ has been applied to ECG in different ways. Cohen et al [6] have treated a set of time domain parameters of the ECG waveform as a vector and applied vector quantisation. The parameters are the amplitude, position and width of the constituent waves (P, QRS and T). The technique in [7] attempted multi-channel ECG compression by applying classified vector quantization (CVQ) to the m-AZTEC parameters. The m-AZTEC is an extension of the AZTEC technique which was proposed for single channel ECG data [8]. In each of the above techniques, the vectors were formed in different ways. However, these techniques were not very successful, as the feature vectors selected as input to the VQ, did not represent the ECG efficiently.

The technique in [4] performs VQ on period and amplitude normalised (PAN) vectors obtained from the variable length ECG beats. As the variation in the beat periods is accounted for by the normalisation, inter and intra-cycle correlations are exploited effectively. In their approach, the signal from one R-wave to the next R-wave is considered as one ECG beat. To get beats of constant length from the varying length ECG cycles, the sampling rate of each of the beats is altered. Fig. 1 illustrates the period normalisation of ECG beats. Each ECG cycle is upsampled by a global factor (N) and downsampled by the respective beat lengths (M_i) so that we get beats of constant length (N). This is followed by amplitude normalisation of each beat, independently. Fig. 2 illustrates the period and amplitude normalisation for a segment of ECG.

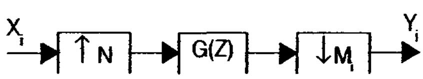

Figure 1: Period normalisation using multirate technique. X_i, Y_i are the i^{th} ECG and PN beats, N is the upsampling factor and M_i is the downsampling factor (length of X_i).

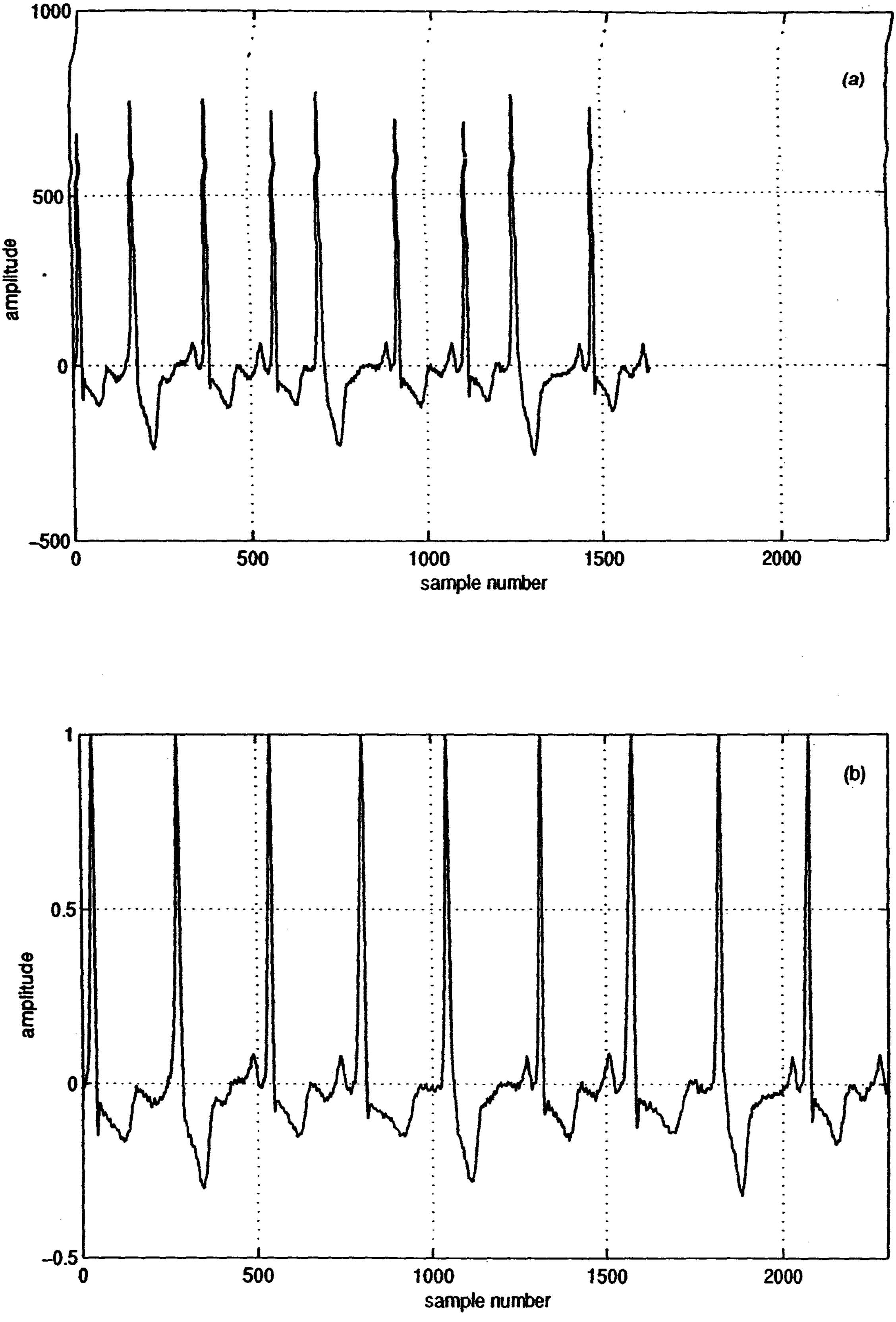

Figure 2: Period and Amplitude Normalisation. (a) A segment of ECG. (b) PAN beats.

Each PAN vector is considered as a single entity and is input to the VQ coder. The PAN-VQ scheme gives a high compression ratio (of 1:200) with an acceptable level of normalised mean square error. However, the utilisation of PAN-VQ for real time ECG compression is limited by its encoding complexity which increases with the dimension of the PAN vector. In the work reported in [4], the dimension of the PAN vector was 256, which is quite large. Thus it is important to reduce the computational complexity of PAN-VQ for realising its full potential. This work extends the PAN-VQ scheme, by proposing a wavelet-VQ coder to reduce the encoding time of the PAN-VQ coder.

The basic VQ algorithm performs a full search through the codebook, which is computationally intense. Many researchers have proposed fast encoding algorithms to accelerate the encoding process. One of such effective coders is the tree structured VQ [9]. It uses a tree-structured codebook and requires only logarithmic encoding time when compared to the full search vector quantizer. A simple technique for handling an unmanageably large task is to decompose it into sub-tasks. Product-code vector quantizer (PCVQ) uses this idea and decomposes the vector into sub-vectors. Instead of using a single codebook, each sub-vector is separately encoded with its own codebook [9]. These methods generally compromise the performance achievable with unconstrained VQ, but often provide very useful and favourable trade-offs between performance and complexity.

In this paper, a fast VQ coder using a product codebook is proposed. Discrete time wavelet transform (DTWT) is used to decompose the PAN-vector into sub-vectors which are approximately independent of one another. The set of wavelet coefficients at each scale is treated as a separate entity and each such sub-vector is encoded with a codebook at the same scale. Since the QRS zone contains important information, a distortion function which gives more weightage to the QRS coefficients is used as the distortion metric. This wavelet vector quantizer (WVQ) reduces the complexity, while paying a small penalty
reduces the complexity, while paying a small penalty
in average distortion.

II. Wavelet-VQ Coder

The PAN beat, which was considered as a single vector in PAN-VQ, is partitioned into sub-vectors. Indices corresponding to the sub-vectors are sent to the decoder. The decoder reconstructs the original vector by first decoding each sub-vector and then combining these vectors to produce the approximation of the original PAN beat. While partitioning a PAN beat, if the sub-vectors are independent of one another, then the coding complexity drastically reduces without a substantial compromise in performance. This motivates us to apply VQ in the transform domain.

In any ECG waveform, the QRS complex is a very well localised, high-frequency region; the P and T waves are low frequency components; the PQ and TP segments are nearly isoelectric with no clinically useful information and the ST segment is a very low frequency, well time localised component. The Fourier transform cannot efficiently characterise the ECG, since such time-localised components affect the entire spectrum. Coding schemes using conventional transforms (such as KLT, DCT or DFT) can perform better only when the signal is stationary, and further when the energy is concentrated in certain bands only. Thus, understandably, all transform-based ECG coding schemes reported earlier have not performed well. However, for efficient representation of such non-stationary signals, wavelets have a lot of potential, since the basic functions have compact support and are of different bandwidths. The nature of ECG, being highly non-stationary within each beat, lends itself very well to wavelet transform-based coding.

A. *Signal decomposition by wavelet transform*

The wavelet transform decomposes a signal x(t) into a weighted sum of basis functions, which are dilated and translated versions of a prototype function called the mother wavelet $\psi(t)$ [10].

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d_{m,k}\, \psi_{m,k}(t) \quad m,k \in Z \qquad (1)$$

$$\psi_{m,k}(t) = 2^{-m/2}\, \psi(2^{-m} t - k) \qquad (2)$$

$$d_{m,k} = \int_{\infty}^{0} x(t)\, \psi_{m,k}(t)\, dt \qquad (3)$$

Here, the value of m controls the scale at which
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the signal is analysed.

The wavelet transform for a discrete signal can be realised by a FIR perfect reconstruction quadrature mirror filter (QMF) bank [11]. Fig. 3 shows a two-channel, FIR perfect reconstruction system with power symmetric filters. If the wavelet bases are orthonormal,

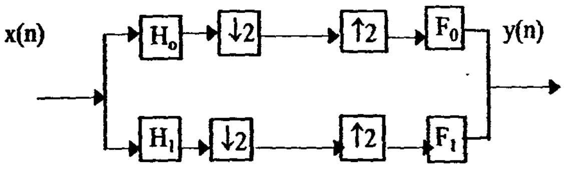

Figure 3: Two channel FIR perfect reconstruction system with power symmetric filters.

then the filters should satisfy the paraunitary condition [11],

$$| H_0(e^{jw}) | + | H_0(e^{j(\pi-w)})^2 | = 1 \tag{4}$$

$$h_1(n) = (-1)^n h_0^*(L-n) \tag{5}$$

$$f_0(n) = h_0^*(L-n) \tag{6}$$

$$f_1(n) = h_1(L-n); \tag{7}$$

where h_0, h_1, f_0, f_1 are the low pass analysis filter, high pass analysis filter, low-pass synthesis filter and high-pass synthesis filter, respectively, L is the length of filter h_0 and H_0 is the fourier transform of h_0.

DTWT coefficients of lower scales can be obtained by using a tree structured filter bank. If each level of the tree is paraunitary, the whole structure is paraunitary, and the wavelet basis realised from this structure satisfies the orthonormality condition. Fig. 4 illustrates the realisation of DTWT by using a tree structured filter bank.

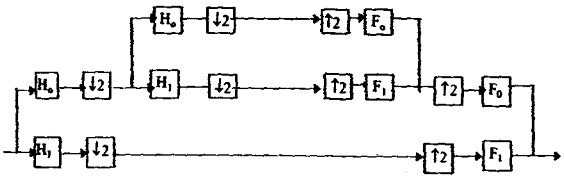

Figure 4: The forward and inverse discrete time wavelet transform implemented using two-level, paraunitary tree structured filter bank.

B. *Encoding and Decoding*

The PAN vector is broken down into coarse and detail signals, by applying wavelet transform. Separate codebooks are generated for different scales and are made available both at the encoder and at the decoder. Most of the ECG energy (and hence the pattern) is preserved in the coefficients corresponding to the coarse approximations. So a larger codebook size is allotted to it.

In PAN-VQ, the conventional mean square error is used as the distortion metric for generating the codebook. As QRS zone contains important information in ECG, we employ weighted mean square error distortion metric, which gives more emphasis to the QRS zone while generating the codebook. If the vectors are of dimension K, then the weighted mean square distortion can be expressed as

$$d^2(X, Y) = \frac{1}{K} [X - Y]^T . W . [X - Y] \tag{8}$$

where W is the weight matrix of size K*K. The MSE distortion is a special case when W = I, the identity matrix. W is so chosen that the QRS zone gets more weightage while generating the codebook. The encoder generates a set of indices corresponding to the best matches in the respective codebooks of each sub-vector.

On receiving the transmitted indices, the decoder outputs the corresponding codebook sub-vectors. These vectors are properly combined to get back the transformed PAN beat. Inverse wavelet transform is applied to this beat to get back the original PAN beat. The difference between the original period and average period having been transmitted to the decoder, the actual period of each beat is obtained the difference between the amplitude and the average amplitude having been transmitted to the decoder, the actual amplitude of the beat is calculated. The PAN beat is multiplied by the actual amplitude and period denormalised to get the reconstructed beats. The block diagrams of the encoder and the decoder of our wavelet-VQ scheme are shown in Fig. 5 and Fig. 6 respectively.

III. Results

The proposed method was tested using ECG data from two different sets of data obtained from the National Institute of Mental Health and Neurosciences, Bangalore. The first set of data was sampled at 250 Hz, whereas the other had a sampling rate of 500 Hz. Both had been quantized to 12 bit resolution. We normalised the number of samples in each beat to 256 (512), although any other number greater than 256 (512) could have been as good, since the total number of samples in the original beats was found to lie between 120 (330) to 240 (510) for the sampling frequency of 250 (500) Hz. Thus, downsampling after interpolation by such a factor, for a highly correlated signal like ECG, does not result in any loss.

To evaluate the performance of the wavelet-VQ, coder, we calculate the compression ratio,

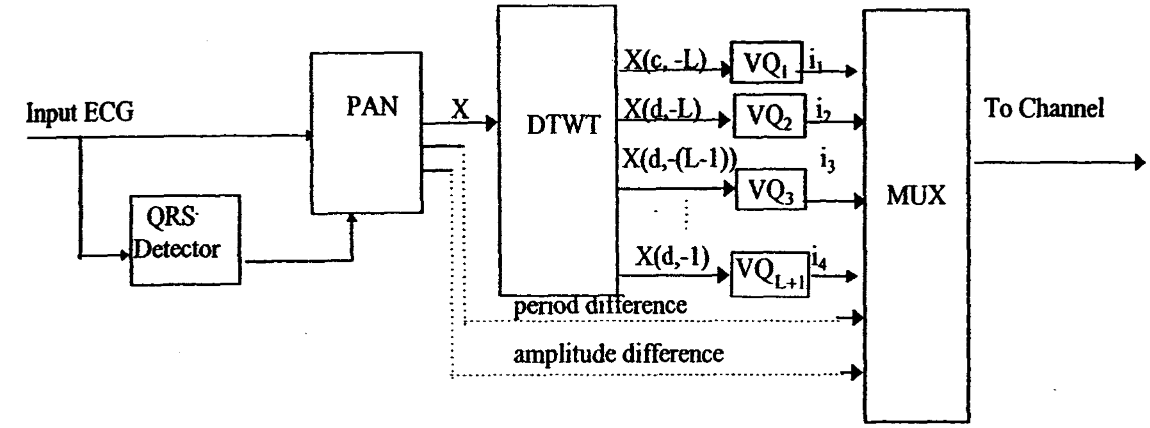

Figure 5: Schematic diagram of wavelet-VQ encoder, X is the input PAN beat, X(c, -L) is the coarse component of X at resolution level L, X(d, -L), X(d, -(L-1)),.... X(d,-1) are the detail components of X.

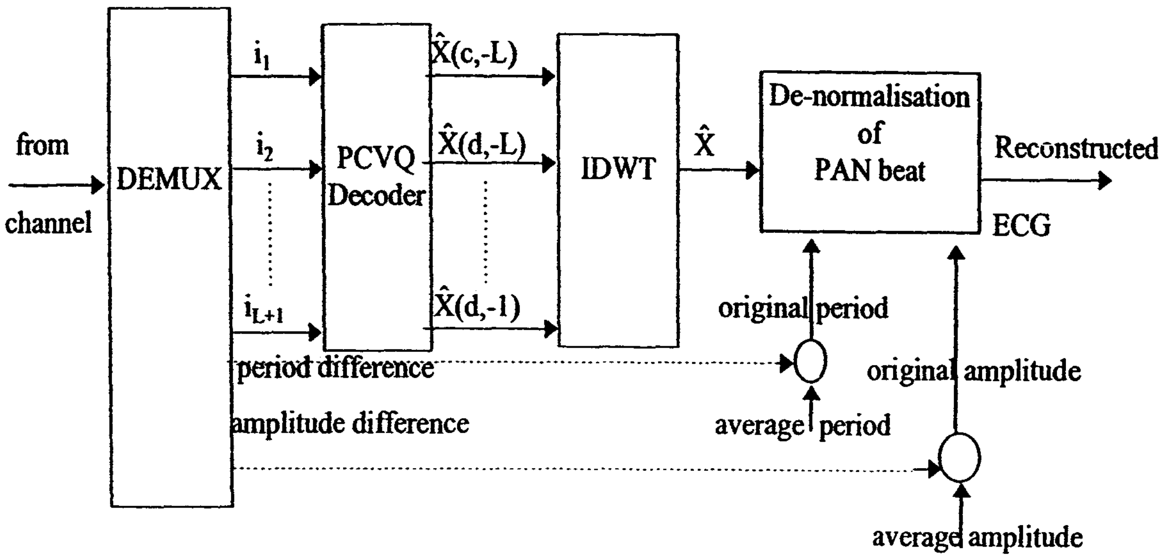

Figure 6: Schematic diagram of the wavelet-VQ decoder, $\hat{X}$ (c,-L),...., $\hat{X}$ (d,-L), $\hat{X}$ (d,-(L-1)), $\hat{X}$ (d,-1) are the reconstructed wavelet components, $\hat{X}$ is the reconstructed PAN beat.

computational complexity and two error measures as discussed below.

A. *Compression ratio:*

$$CR = \frac{b_0 \sum_{i=1}^{N_T} T_i}{N_T\left(\sum_{i=1}^{L+1} \log_2 S_i + b_1 + b_2\right)} \qquad (9)$$

where N_T is the total number of beats transmitted, T_i is the original period of the i^{th} beat, S_i is the size of the codebook at the i^{th} resolution level, L is the total number of levels the signal is decomposed into, b_0, b_1, b_2 are the number of bits used to quantize the digitised ECG samples, the amplitude difference, and the period difference, respectively.

The expression in our case reduces to

$$CR = \frac{12 \sum_{i=1}^{N} T_i}{N_T\left(\sum_{i=1}^{3} \log_2 S_i + 12\right)} \qquad (10)$$

The codebook sizes used are given in Table I.

C. *Normalised Maximum Amplitude Error (NMAE) :*

The maximum amplitude of error in each cycle is normalised by the dynamic range of the original cycle. The expression for NMAE for the i^{th} cycle is

$$NMAE_i = \frac{\max |X_{io} - X_{ir}|}{\max X_{io} - \min X_{io}} \qquad (12)$$

where X_{io} and X_{ir} refer to the i^{th} original and reconstructed beat vectors respectively and max X_{io} and min X_{io} are the maximum and the minimum values of the vector X_{io} respectively. This is averaged over all the cycles to get the mean NMAE for each subject.

D. *Computational Complexity*

To compare the computational complexity of the encoder with that of the normal VQ, we assume 2 additions and one subtraction as one computational unit (CU). If K is the dimension of the vector and N is the size of the codebook, the computational complexity (CC) of direct VQ is N*K CUs. To keep the encoder computational complexity of waveiet-VQ at a minimum, we used *Haar wavelet*. The number of CUs required for a L level decomposition using Haar is

$K + \frac{K}{2} + ... + \frac{K}{2^L}$. Thus the total complexity of the encoder is given by

$$CC = K + \frac{K}{2} + ... \frac{K}{2^L} + S_1 * \frac{K}{2^L} + S_2 * \frac{K}{2^L} + S_3 * \frac{K}{2^{L-1}} + ...$$

$$+ S_{L+1} * \frac{K}{2} \text{ CUs} \qquad (13)$$

where S_1 is the size of the coarse component codebook and S_2, S_3... S_{L+1} are the sizes of the codebooks for the detail components of DTWT of the PAN beat. In our trials, we performed wavelet decomposition only upto 2 levels. Table I gives the codebook sizes for the coarse and detail signals.

We have achieved compression ratios of 100 to 200 depending on the size of the codebook used. Tables II and III give the figures for the CR, CC, NRMSE and NMAE for 6 different subjects using PAN-VQ and wavelet-VQ (for different codebook sizes). Table III shows the performance figures with different sampling rates. The NRMSE values achieved by our technique are better than those of PAN-VQ, while the complexity is reduced to half of that. Fig. 7 shows the original, reconstructed and error waveforms for a subject sampled at 500 Hz.

Table 1 : Codebook sizes used for the wavelet-VQ coder

	coarse S(1)	Detail-1 S(2)	detail-2 S(3)
Set-1	32	16	16
Set-2	32	8	8

Table 2 : Performance figures of wavelet-VQ coder for 3 different subjects. (sampling rate of 500 hz)

	CR	CC	NRMSE	NMAE
PAN-VQ	170	8192	8.26	3.24
W-VQ(32-16-16)	115	5504	7.07	2.88
W-VQ(32-8-8)	125	4480	8.14	3.33

Table 3 : Performance figures of wavelet-VQ coder for 3 different subjects. (sampling rate of 250 Hz.)

	CR	CC	NRMSE	NMAE
PAN-VQ	300	16384	9.41	4.21
W-VQ(32-16-16)	200	13056	8.32	3.08
W-VQ(32-8-8)	230	8960	9.28	3.97

IV. Conclusions

A novel, fast VQ coder for compressing the ECG data is proposed in this paper. The technique gives a good compression ratio with an acceptable fidelity of reconstruction. Keeping this advantage, the proposed coder provides a favourable trade-off between the computational complexity and the performance of the VQ. The proposed coder achieves a factor of two reduction in the number of computations while paying a small penalty in the performance and can be implemented in real time on DSP processor kits.

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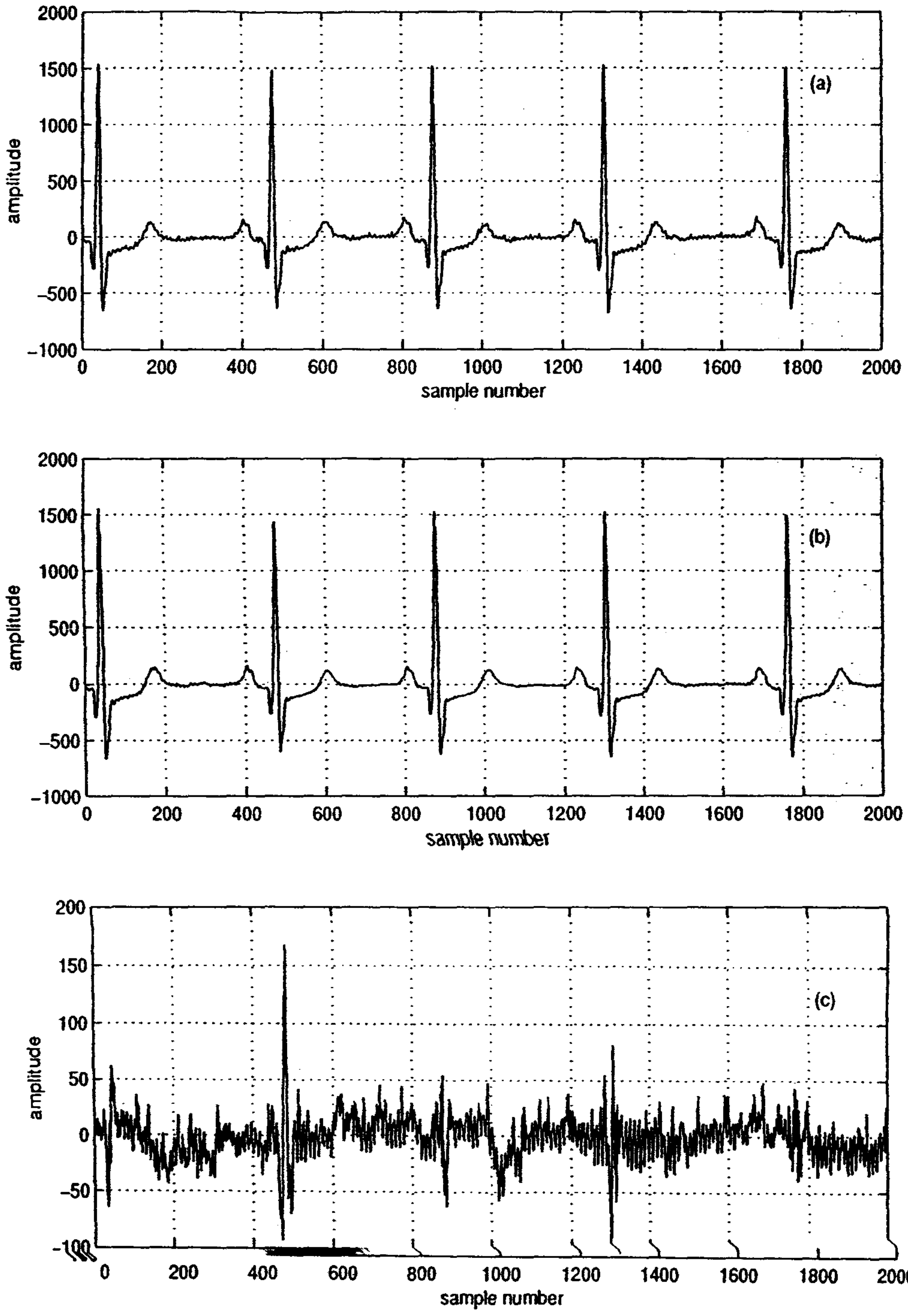

Figure 7: Results of wavelet-VQ on a subject. (a) Original ECG (b) Reconstructed signal (c) Reconstruction Error.

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